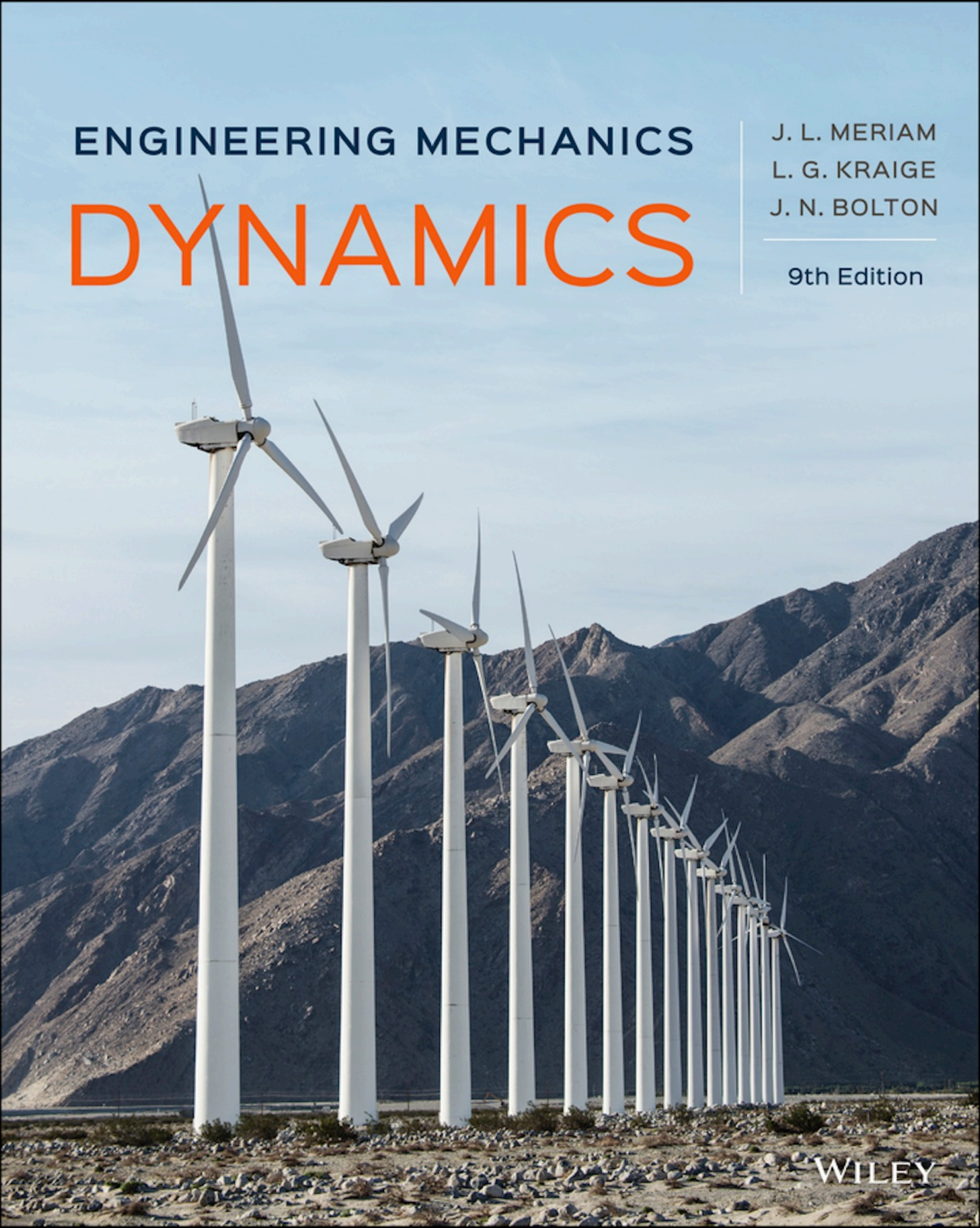

ENGINEERING MECHANICS

DYNAMICS

J. L. MERIAM
L. G. KRAIGE
J. N. BOLTON

9th Edition

WILEY

Engineering Mechanics

Volume 2

Dynamics

Ninth Edition

Engineering Mechanics

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J.L. MERIAM

L.G. KRAIGE

Virginia Polytechnic Institute and State University

J.N. BOLTON

Bluefield State College

WILEY

On the cover: The principles of dynamics must be applied during the design of wind turbines.

VICE PRESIDENT AND EXECUTIVE PUBLISHER: Don Fowley
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COVER PHOTO: © Mint Images/Getty Images, Inc.

This book was set in 9.5/11.5 New Century Schoolbook Lt Std. by Aptara, Inc., and printed by Quad Graphics. The cover was printed by Quad Graphics.

This book is printed on acid-free paper. ∞

Library of Congress Cataloging-in-Publication Data

Names: Meriam, J. L. (James L.), author. | Kraige, L. G. (L. Glenn), author.
| Bolton, J. N. (Jeffrey N.), author.

Title: Dynamics / by J.L. Meriam, L.G. Kraige, J.N. Bolton.

Description: Ninth edition. | Hoboken, NJ : Wiley, [2018] | Series:
Engineering mechanics ; volume 2 | Includes bibliographical references and index. |

Identifiers: LCCN 2017043476 (print) | LCCN 2017045670 (ebook) | ISBN
9781119390985 (enhanced epub) | ISBN | ISBN 9781119390985 (ePub)

Subjects: LCSH: Machinery, Dynamics of.
Classification: LCC TA352 (ebook) | LCC TA352 .M45 2018 (print) | DDC
620.1--dc23

LC record available at https://lccn.loc.gov/2017043476

ISBN: 978-1119-39098-5

10 9 8 7 6 5 4 3 2 1

Foreword

This series of textbooks was begun in 1951 by the late Dr. James L. Meriam. At that time, the books represented a revolutionary transformation in undergraduate mechanics education. They became the definitive textbooks for the decades that followed as well as models for other engineering mechanics texts that have subsequently appeared. Published under slightly different titles prior to the 1978 First Editions, this textbook series has always been characterized by logical organization, clear and rigorous presentation of the theory, instructive sample problems, and a rich collection of real-life problems, all with a high standard of illustration. In addition to the U.S. versions, the books have appeared in SI versions and have been translated into many foreign languages. These textbooks collectively represent an international standard for undergraduate texts in mechanics.

The innovations and contributions of Dr. Meriam (1917–2000) to the field of engineering mechanics cannot be overstated. He was one of the premier engineering educators of the second half of the twentieth century. Dr. Meriam earned the B.E., M.Eng., and Ph.D. degrees from Yale University. He had early industrial experience with Pratt and Whitney Aircraft and the General Electric Company. During the Second World War he served in the U.S. Coast Guard. He was a member of the faculty of the University of California—Berkeley, Dean of Engineering at Duke University, a faculty member at the California Polytechnic State University, and visiting professor at the University of California—Santa Barbara, finally retiring in 1990. Professor Meriam always placed great emphasis on teaching, and this trait was recognized by his students wherever he taught. He was the recipient of several teaching awards, including the Benjamin Garver Lamme Award, which is the highest annual national award of the American Society of Engineering Education (ASEE).

Dr. L. Glenn Kraige, coauthor of the *Engineering Mechanics* series since the early 1980s, has also made significant contributions to mechanics education. Dr. Kraige earned his B.S., M.S., and Ph.D. degrees at the University of Virginia, principally in aerospace engineering, and he is Professor Emeritus of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University. During the mid-1970s, I had the singular pleasure of chairing Professor Kraige's graduate committee and take particular pride in the fact that he was the first of my fifty-four Ph.D. graduates. Professor Kraige was invited by Professor Meriam to team with him, thereby ensuring that the Meriam legacy of textbook authorship excellence would be carried forward to future generations of engineers.

In addition to his widely recognized research and publications in the field of spacecraft dynamics, Professor Kraige has devoted his attention to the teaching of mechanics at both introductory and advanced levels. His outstanding teaching has been widely recognized and has earned him teaching awards at the departmental, college, university, state, regional, and national levels. These awards include the Outstanding Educator Award from the State Council of Higher Education for the Commonwealth of Virginia. In 1996, the Mechanics Division of ASEE bestowed upon him the Archie Higdon Distinguished Educator Award. The Carnegie Foundation for the Advancement of Teaching and the Council for Advancement and Support of Education awarded him the distinction of Virginia Professor of the Year for 1997. In his teaching, Professor Kraige stresses the development of analytical capabilities along with the strengthening of physical insight and engineering judgment. Since the early 1980s, he has worked on personal-computer software designed to enhance the teaching/learning process in statics, dynamics, strength of materials, and higher-level areas of dynamics and vibrations.

Continuing as coauthor for this edition is Dr. Jeffrey N. Bolton, Associate Professor of Mechanical Engineering Technology and Director of Digital Learning at Bluefield State College. Dr. Bolton earned his B.S., M.S., and Ph.D. in Engineering Mechanics from Virginia Polytechnic Institute and State University. His research interests include automatic balancing of six-degree-of-freedom elastically-mounted rotors. He has a wealth of teaching experience, including at Virginia Tech, where he was the 2010 recipient of the Sporn Teaching Award for Engineering Subjects, which is primarily chosen by students. In 2014, Professor Bolton received the Outstanding Faculty Award from Bluefield State College. Professor Bolton was selected as the 2016 West Virginia Professor of the Year by the Faculty Merit Foundation. He has the unusual ability to set high levels of rigor and achievement in the classroom while establishing a high degree of rapport with his students. In addition to maintaining time-tested traditions for future generations of students, Dr. Bolton brings effective application of technology to this textbook series.

The Ninth Edition of *Engineering Mechanics* continues the same high standards set by previous editions and adds new features of help and interest to students. It contains a vast collection of interesting and instructive problems. The faculty and students privileged to teach or study from the Meriam/Kraige/Bolton *Engineering Mechanics* series will benefit from several decades of investment by three highly accomplished educators. Following the pattern of the previous editions, this textbook stresses the application of theory to actual engineering situations, and at this important task it remains the best.

John L. Junkins

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Preface

Engineering mechanics is both a foundation and a framework for most of the branches of engineering. Many of the topics in such areas as civil, mechanical, aerospace, and agricultural engineering, and of course engineering mechanics itself, are based upon the subjects of statics and dynamics. Even in a discipline such as electrical engineering, practitioners, in the course of considering the electrical components of a robotic device or a manufacturing process, may find themselves first having to deal with the mechanics involved.

Thus, the engineering mechanics sequence is critical to the engineering curriculum. Not only is this sequence needed in itself, but courses in engineering mechanics also serve to solidify the student's understanding of other important subjects, including applied mathematics, physics, and graphics. In addition, these courses serve as excellent settings in which to strengthen problem-solving abilities.

Philosophy

The primary purpose of the study of engineering mechanics is to develop the capacity to predict the effects of force and motion while carrying out the creative design functions of engineering. This capacity requires more than a mere knowledge of the physical and mathematical principles of mechanics; also required is the ability to visualize physical configurations in terms of real materials, actual constraints, and the practical limitations which govern the behavior of machines and structures. One of the primary objectives in a mechanics course is to help the student develop this ability to visualize, which is so vital to problem formulation. Indeed, the construction of a meaningful mathematical model is often a more important experience than its solution. Maximum progress is made when the principles and their limitations are learned together within the context of engineering application.

There is a frequent tendency in the presentation of mechanics to use problems mainly as a vehicle to illustrate theory rather than to develop theory for the purpose of solving problems. When the first view is allowed to predominate, problems tend to become overly idealized and unrelated to engineering with the result that the exercise becomes dull, academic, and uninteresting. This approach deprives the student of valuable experience in formulating problems and thus of discovering the need for and meaning of theory. The second view provides by far the stronger motive for learning theory and leads to a better balance between theory and application. The crucial role played by interest and purpose in providing the strongest possible motive for learning cannot be overemphasized.

Furthermore, as mechanics educators, we should stress the understanding that, at best, theory can only approximate the real world of mechanics rather than the view that the real world approximates the theory. This difference in philosophy is indeed basic and distinguishes the *engineering* of mechanics from the *science* of mechanics.

Over the past several decades, several unfortunate tendencies have occurred in engineering education. First, emphasis on the geometric and physical meanings of prerequisite mathematics appears to have diminished. Second, there has been a significant reduction and even elimination of instruction in graphics, which in the past enhanced the visualization and representation of mechanics problems. Third, in advancing the mathematical level of our treatment of mechanics, there has been a tendency to allow the notational manipulation of vector operations to mask or replace geometric visualization. Mechanics is inherently a subject which depends on geometric and physical perception, and we should increase our efforts to develop this ability.

A special note on the use of computers is in order. The experience of formulating problems, where reason and judgment are developed, is vastly more important for the student than is the manipulative exercise in carrying out the solution. For this reason, computer usage must be carefully controlled. At present, constructing free-body diagrams and formulating governing equations are best done with pencil and paper. On the other hand, there are instances in which the *solution* to the governing equations can best be carried out and displayed using the computer. Computer-oriented problems should be genuine in the sense that there is a condition of design or criticality to be found, rather than "makework" problems in which some parameter is varied for no apparent reason other than to force artificial use of the computer. These thoughts have been kept in mind during the design of the computer-oriented problems in the

Ninth Edition. To conserve adequate time for problem formulation, it is suggested that the student be assigned only a limited number of the computer-oriented problems.

As with previous editions, this Ninth Edition of *Engineering Mechanics* is written with the foregoing philosophy in mind. It is intended primarily for the first engineering course in mechanics, generally taught in the second year of study. *Engineering Mechanics* is written in a style which is both concise and friendly. The major emphasis is on basic principles and methods rather than on a multitude of special cases. Strong effort has been made to show both the cohesiveness of the relatively few fundamental ideas and the great variety of problems which these few ideas will solve.

Organization

The logical division between particle dynamics (Part I) and rigid-body dynamics (Part II) has been preserved, with each part treating the kinematics prior to the kinetics. This arrangement promotes thorough and rapid progress in rigid-body dynamics with the prior benefit of a comprehensive introduction to particle dynamics.

In Chapter 1, the fundamental concepts necessary for the study of dynamics are established.

Chapter 2 treats the kinematics of particle motion in various coordinate systems, as well as the subjects of relative and constrained motion.

Chapter 3 on particle kinetics focuses on the three basic methods: force-mass-acceleration (Section A), work-energy (Section B), and impulse-momentum (Section C). The special topics of impact, central-force motion, and relative motion are grouped together in a special applications section (Section D) and serve as optional material to be assigned according to instructor preference and available time. With this arrangement, the attention of the student is focused more strongly on the three basic approaches to kinetics.

Chapter 4 on systems of particles is an extension of the principles of motion for a single particle and develops the general relationships which are so basic to the modern comprehension of dynamics. This chapter also includes the topics of steady mass flow and variable mass, which may be considered as optional material.

In Chapter 5 on the kinematics of rigid bodies in plane motion, where the equations of relative velocity and relative acceleration are encountered, emphasis is placed jointly on solution by vector geometry and solution by vector algebra. This dual approach serves to reinforce the meaning of vector mathematics.

In Chapter 6 on the kinetics of rigid bodies, we place great emphasis on the basic equations which govern all categories of plane motion. Special emphasis is also placed on forming the direct equivalence between the actual applied forces and couples and their $m\bar{a}$ and $\bar{I}\alpha$ resultants. In this way the versatility of the moment principle is emphasized, and the student is encouraged to think directly in terms of resultant dynamics effects.

Chapter 7, which may be treated as optional, provides a basic introduction to three-dimensional dynamics which is sufficient to solve many of the more common space-motion problems. For students who later pursue more advanced work in dynamics, Chapter 7 will provide a solid foundation. Gyroscopic motion with steady precession is treated in two ways. The first approach makes use of the analogy between the relation of force and linear-momentum vectors and the relation of moment and angular-momentum vectors. With this treatment, the student can understand the gyroscopic phenomenon of steady precession and can handle most of the engineering problems on gyroscopes without a detailed study of three-dimensional dynamics. The second approach employs the more general momentum equations for three-dimensional rotation where all components of momentum are accounted for.

Chapter 8 is devoted to the topic of vibrations. This full-chapter coverage will be especially useful for engineering students whose only exposure to vibrations is acquired in the basic dynamics course.

Moments and products of inertia of mass are presented in Appendix B. Appendix C contains a summary review of selected topics of elementary mathematics as well as several numerical techniques which the student should be prepared to use in computer-solved problems. Useful tables of physical constants, centroids, moments of inertia, and conversion factors are contained in Appendix D.

Pedagogical Features

The basic structure of this textbook consists of an article which rigorously treats the particular subject matter at hand, followed by one or more sample problems. For the Ninth Edition, all homework problems have been moved to a special Student Problems section found after Appendix D near the end of the textbook. There is a Chapter Review at the end of each chapter which summarizes the main points in that chapter, and a corresponding Chapter Review Problem set found in the Student Problems section.

Problems

The 124 Sample Problems appear on specially designed pages by themselves. The solutions to typical dynamics problems are presented in detail. In addition, explanatory and cautionary notes (Helpful Hints) are number-keyed to the main presentation.

There are 1277 homework exercises. The problem sets are divided into *Introductory Problems* and *Representative Problems*. The first section consists of simple, uncomplicated problems designed to help students gain confidence with the new topic, while most of the problems in the second section are of average difficulty and length. The problems are generally arranged in order of increasing difficulty. More difficult exercises appear near the end of the *Representative Problems* and are marked with the triangular symbol ▶. *Computer-Oriented Problems*, marked with an asterisk, appear throughout the problems and also in a special section at the conclusion of the Chapter Review Problems. Problems marked with the student-solution icon SS have solutions available in the Enhanced eText and WileyPLUS. The answers to all problems have been provided in a special section near the end of the textbook.

In recognition of the need for emphasis on SI units, there are approximately two problems in SI units for every one in U.S. customary units. This apportionment between the two sets of units permits anywhere from a 50–50 emphasis to a 100-percent SI treatment.

A notable feature of the Ninth Edition, as with all previous editions, is the wealth of interesting and important problems which apply to engineering design. Whether directly identified as such or not, virtually all of the problems deal with principles and procedures inherent in the design and analysis of engineering structures and mechanical systems.

Illustrations

In order to bring the greatest possible degree of realism and clarity to the illustrations, the electronic version of this textbook series continues to be produced in full color. It is important to note that color is used consistently for the identification of certain quantities:

- *red* for forces and moments
- *green* for velocity and acceleration arrows
- *orange dashes* for selected trajectories of moving points

Subdued colors are used for those parts of an illustration which are not central to the problem at hand. Whenever possible, mechanisms or objects which commonly have a certain color will be portrayed in that color. All of the fundamental elements of technical illustration which have been an essential part of this *Engineering Mechanics* series of textbooks have been retained. The authors wish to restate the conviction that a high standard of illustration is critical to any written work in the field of mechanics.

Special Features

We have retained the following hallmark features of previous editions:

- The main emphasis on the work-energy and impulse-momentum equations is on the time-order form, both for particles in Chapter 3 and rigid bodies in Chapter 6.
- Emphasis has been placed on three-part impulse-momentum diagrams, both for particles and rigid bodies. These diagrams are well integrated with the time-order form of the impulse-momentum equations.
- Within-the-chapter photographs are provided in order to provide additional connection to actual situations in which dynamics has played a major role.
- All Sample Problems are printed on specially designed pages for quick identification.
- All theory portions have been reexamined in order to maximize rigor, clarity, readability, and level of friendliness.
- Key Concepts areas within the theory presentation have been specially marked and highlighted.
- The Chapter Reviews are highlighted and feature itemized summaries.

Resources and Formats

The following items have been prepared to complement this textbook:

Instructor and Student Resources

The following resources are available online at *www.wiley.com/college/meriam*. There may be additional resources not listed.

WileyPLUS: A complete online learning system to help prepare and present lectures, assign and manage homework, keep track of student progress, and customize your course content and delivery. Newly added materials for *WileyPLUS* include step-by-step video solutions for approximately 225 problems, all of which are similar to those found in the textbook. These author-generated videos illustrate clear and concise solution strategies for students, further emphasizing key concepts and demonstrating sound principles of problem solving in mechanics.

Instructor's Manual: Prepared by the authors and independently checked, fully worked solutions to all problems in the text are available to faculty by contacting their local Wiley representative.

All ***figures*** in the text are available in electronic format for use in creating lecture presentations.

All ***Sample Problems*** are available as electronic files for display and discussion in the classroom.

Formats

This Ninth Edition is available in a variety of formats, including conventional print, WileyPLUS standalone, standalone alone e-text (now with numerous enhancements), and other bundled formats. Please contact a Wiley representative (www.wiley.com/go/whosmyrep) for more information.

Acknowledgments

Special recognition is due Dr. A. L. Hale, formerly of Bell Telephone Laboratories, for his continuing contribution in the form of invaluable suggestions and accurate checking of the manuscript. Dr. Hale has rendered similar service for all previous versions of this entire series of mechanics books, dating back to the early 1950s. He reviews all aspects of the books, including all old and new text and figures. Dr. Hale carries out an independent solution to each new homework exercise and provides the authors with suggestions and needed corrections to the solutions which appear in the *Instructor's Manual*. Dr. Hale is well known for being extremely accurate in his work, and his fine knowledge of the English language is a great asset which aids every user of this textbook.

We would like to thank the faculty members of the Department of Engineering Science and Mechanics at VPI&SU who regularly offer constructive suggestions. These include Saad A. Ragab, Norman E. Dowling, Michael W. Hyer (late), J. Wallace Grant, and Jacob Grohs. Scott L. Hendricks has been particularly effective and accurate in his extensive review of the manuscript and preparation of WileyPLUS materials. Michael Goforth of Bluefield State College is acknowledged for his significant contribution to supplementary textbook materials. Nathaniel Greene of Bloomfield State University of Pennsylvania is recognized for his careful reading and suggestions for improvement.

The contributions by the staff of John Wiley & Sons, Inc., reflect a high degree of professional competence and are duly recognized. These include Executive Editor Linda Ratts, Associate Developmental Editor Adria Giattino, Editorial Assistant Adriana Alecci, Senior Production Editor Ken Santor, Senior Designer Wendy Lai, and Senior Photograph Editor Billy Ray. We wish to especially acknowledge the long-term production efforts of Christine Cervoni of Camelot Editorial Services, LLC, as well as the editing of Helen Walden. The talented illustrators of Lachina continue to maintain a high standard of illustration excellence.

Finally, we wish to state the extremely significant contribution of our families for the patience and support over the many hours of manuscript preparation. In particular, Dale Kraige has managed the preparation of the manuscript for the Ninth Edition and has been a key individual in checking all stages of the proof.

We are extremely pleased to participate in extending the time duration of this textbook series well past the sixty-five-year mark. In the interest of providing you with the best possible educational materials over future years, we encourage and welcome all comments and suggestions.

L. Glenn Kraige

Blacksburg, Virginia
Princeton, West Virginia

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PART I

Dynamics of Particles

CHAPTER 1

Introduction to Dynamics

CHAPTER OUTLINE

NASA

The International Space Station's Canadarm2 grapples the Kounotori2 H-II Transfer Vehicle as it approaches the station in 2011.

1/1 History and Modern Applications

Dynamics is that branch of mechanics which deals with the motion of bodies under the action of forces. The study of dynamics in engineering usually follows the study of statics, which deals with the effects of forces on bodies at rest. Dynamics has two distinct parts: *kinematics*, which is the study of motion without reference to the forces which cause motion, and *kinetics*, which relates the action of forces on bodies to their resulting motions. A thorough comprehension of dynamics will provide one of the most useful and powerful tools for analysis in engineering.

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Galileo Galilei

Portrait of Galileo Galilei (1564–1642) (oil on canvas), Sustermans, Justus (1597–1681) (school of)/ Galleria Palatina, Florence, Italy/Bridgeman Art Library.

History of Dynamics

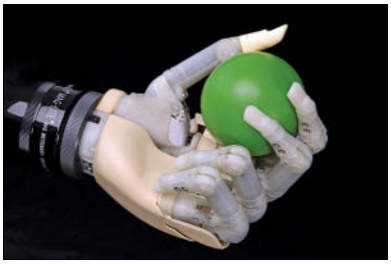
Dynamics is a relatively recent subject compared with statics. The beginning of a rational understanding of dynamics is credited to Galileo (1564–1642), who made careful observations concerning bodies in free fall, motion on an inclined plane, and motion of the pendulum. He was largely responsible for bringing a scientific approach to the investigation of physical problems. Galileo was continually under severe criticism for refusing to accept the established beliefs of his day, such as the philosophies of Aristotle which held, for example, that heavy bodies fall more rapidly than light bodies. The lack of accurate means for the measurement of time was a severe handicap to Galileo, and further significant development in dynamics awaited the invention of the pendulum clock by Huygens in 1657.

Newton (1642–1727), guided by Galileo's work, was able to make an accurate formulation of the laws of motion and, thus, to place dynamics on a sound basis. Newton's famous work was published in the first edition of his *Principia*,* which is generally recognized as one of the greatest of all recorded contributions to knowledge. In addition to stating the laws governing the motion of a particle, Newton was the first to correctly formulate the law of universal gravitation. Although his mathematical description was accurate, he felt that the concept of remote transmission of gravitational force without a supporting medium was an absurd notion. Following Newton's time, important contributions to mechanics were made by Euler, D'Alembert, Lagrange, Laplace, Poinsot, Coriolis, Einstein, and others.

Applications of Dynamics

Only since machines and structures have operated with high speeds and appreciable accelerations has it been necessary to make calculations based on the principles of dynamics rather than on the principles of statics. The rapid technological developments of the present day require increasing application of the principles of mechanics, particularly dynamics. These principles are basic to the analysis and design of moving structures, to fixed structures subject to shock loads, to robotic devices, to automatic control systems, to rockets, missiles, and spacecraft, to ground and air transportation vehicles, to electron ballistics of electrical devices, and to machinery of all types such as turbines, pumps, reciprocating engines, hoists, machine tools, etc.

Students with interests in one or more of these and many other activities will constantly need to apply the fundamental principles of dynamics.



WENN Ltd/Alamy Stock Photo

Artificial hand

1/2 Basic Concepts

The concepts basic to mechanics were set forth in Art. 1/2 of *Vol. 1 Statics*. They are summarized here along with additional comments of special relevance to the study of dynamics.

Space is the geometric region occupied by bodies. Position in space is determined relative to some geometric reference system by means of linear and angular measurements. The basic frame of reference for the laws of Newtonian mechanics is the *primary inertial system* or *astronomical frame of reference*, which is an imaginary set of rectangular axes assumed to have no translation or rotation in space. Measurements show that the laws of Newtonian mechanics are valid for this reference system as long as any velocities involved are negligible compared with the speed of light, which is 300 000 km/s or 186,000 mi/sec. Measurements made with respect to this reference are said to be *absolute*, and this reference system may be considered "fixed" in space.

A reference frame attached to the surface of the earth has a somewhat complicated motion in the primary system, and a correction to the basic equations of mechanics must be applied for measurements made relative to the reference frame of the earth. In the calculation of rocket and space-flight trajectories, for example, the

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*The original formulations of Sir Isaac Newton may be found in the translation of his *Principia* (1687), revised by F. Cajori, University of California Press, 1934.

absolute motion of the earth becomes an important parameter. For most engineering problems involving machines and structures which remain on the surface of the earth, the corrections are extremely small and may be neglected. For these problems the laws of mechanics may be applied directly with measurements made relative to the earth, and in a practical sense such measurements will be considered *absolute*.

Time is a measure of the succession of events and is considered an absolute quantity in Newtonian mechanics.

Mass is the quantitative measure of the inertia or resistance to change in motion of a body. Mass may also be considered as the quantity of matter in a body as well as the property which gives rise to gravitational attraction.

Force is the vector action of one body on another. The properties of forces have been thoroughly treated in *Vol. 1 Statics*.

A ***particle*** is a body of negligible dimensions. When the dimensions of a body are irrelevant to the description of its motion or the action of forces on it, the body may be treated as a particle. An airplane, for example, may be treated as a particle for the description of its flight path.

A ***rigid body*** is a body whose changes in shape are negligible compared with the overall dimensions of the body or with the changes in position of the body as a whole. As an example of the assumption of rigidity, the small flexural movement of the wing tip of an airplane flying through turbulent air is clearly of no consequence to the description of the motion of the airplane as a whole along its flight path. For this purpose, then, the treatment of the airplane as a rigid body is an acceptable approximation. On the other hand, if we need to examine the internal stresses in the wing structure due to changing dynamic loads, then the deformation characteristics of the structure would have to be examined, and for this purpose the airplane could no longer be considered a rigid body.

Vector and ***scalar*** quantities have been treated extensively in *Vol. 1 Statics*, and their distinction should be perfectly clear by now. Scalar quantities are printed in lightface italic type, and vectors are shown in boldface type. Thus, V denotes the scalar magnitude of the vector $\mathbf{V}$. It is important that we use an identifying mark, such as an underline $\underline{V}$, for all handwritten vectors to take the place of the boldface designation in print. For two nonparallel vectors recall, for example, that $\mathbf{V}_1 + \mathbf{V}_2$ and $V_1 + V_2$ have two entirely different meanings.

We assume that you are familiar with the geometry and algebra of vectors through previous study of statics and mathematics. Students who need to review these topics will find a brief summary of them in Appendix C along with other mathematical relations which find frequent use in mechanics. Experience has shown that the geometry of mechanics is often a source of difficulty for students. Mechanics by its very nature is geometrical, and students should bear this in mind as they review their mathematics. In addition to vector algebra, dynamics requires the use of vector calculus, and the essentials of this topic will be developed in the text as they are needed.

Dynamics involves the frequent use of time derivatives of both vectors and scalars. As a notational shorthand, a dot over a symbol will frequently be used to indicate a derivative with respect to time. Thus, $\dot{x}$ means dx/dt and $\ddot{x}$ stands for d^2x/dt^2.

1/3 Newton's Laws

Newton's three laws of motion, stated in Art. 1/4 of *Vol. 1 Statics*, are restated here because of their special significance to dynamics. In modern terminology they are:

> **Law I.** A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.*

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

These laws have been verified by countless physical measurements. The first two laws hold for measurements made in an absolute frame of reference, but are subject to some correction when the motion is measured relative to a reference system having acceleration, such as one attached to the surface of the earth.

Newton's second law forms the basis for most of the analysis in dynamics. For a particle of mass m subjected to a resultant force $\mathbf{F}$, the law may be stated as

$$\boxed{\mathbf{F} = m\mathbf{a}} \qquad (1/1)$$

where $\mathbf{a}$ is the resulting acceleration measured in a nonaccelerating frame of reference. Newton's first law is a consequence of the second law since there is no acceleration when the force is zero, and so the particle is either at rest or is moving with constant velocity. The third law constitutes the principle of action and reaction with which you should be thoroughly familiar from your work in statics.

1/4 Units

Both the International System of metric units (SI) and the U.S. customary system of units are defined and used in *Vol. 2 Dynamics*, although a stronger emphasis is placed on the metric system because it is replacing the U.S. customary system. However, numerical conversion from one system to the other will often be needed in U.S. engineering practice for some years to come. To become familiar with each system, it is necessary to think directly in that system. Familiarity with the new system cannot be achieved simply by the conversion of numerical results from the old system.

Tables defining the SI units and giving numerical conversions between U.S. customary and SI units are included in Table D/5 of Appendix D.

The four fundamental quantities of mechanics, and their units and symbols for the two systems, are summarized in the following table:

Quantity	Dimensional Symbol		SI Units			U.S. Customary Units	
			Unit	Symbol		Unit	Symbol
Mass	M	Base units	kilogram	kg		slug	—
Length	L		meter*	m	Base units	foot	ft
Time	T		second	s		second	sec
Force	F		newton	N		pound	lb

*Also spelled *metre*.

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*To some it is preferable to interpret Newton's second law as meaning that the resultant force acting on a particle is proportional to the time rate of change of momentum of the particle and that this change is in the direction of the force. Both formulations are equally correct when applied to a particle of constant mass.

As shown in the table, in SI the units for mass, length, and time are taken as base units, and the units for force are derived from Newton's second law of motion, Eq. 1/1. In the U.S. customary system the units for force, length, and time are base units and the units for mass are derived from the second law.

The SI system is termed an *absolute* system because the standard for the base unit kilogram (a platinum-iridium cylinder kept at the International Bureau of Standards near Paris, France) is independent of the gravitational attraction of the earth. On the other hand, the U.S. customary system is termed a *gravitational* system because the standard for the base unit pound (the weight of a standard mass located at sea level and at a latitude of 45°) requires the presence of the gravitational field of the earth. This distinction is a fundamental difference between the two systems of units.

In SI units, by definition, one newton is that force which will give a one-kilogram mass an acceleration of one meter per second squared. In the U.S. customary system a 32.1740-pound mass (1 slug) will have an acceleration of one foot per second squared when acted on by a force of one pound. Thus, for each system we have from Eq. 1/1


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The U.S. standard kilogram at the National Bureau of Standards.

SI Units	U.S. Customary Units
$(1\text{ N}) = (1\text{ kg})(1\text{ m/s}^2)$	$(1\text{ lb}) = (1\text{ slug})(1\text{ ft/sec}^2)$
$\text{N} = \text{kg}\cdot\text{m/s}^2$	$\text{slug} = \text{lb}\cdot\text{sec}^2/\text{ft}$

In SI units, the kilogram should be used *exclusively* as a unit of mass and *never* force. Unfortunately, in the MKS (meter, kilogram, second) gravitational system, which has been used in some countries for many years, the kilogram has been commonly used both as a unit of force and as a unit of mass.

In U.S. customary units, the pound is unfortunately used both as a unit of force (lbf) and as a unit of mass (lbm). The use of the unit lbm is especially prevalent in the specification of the thermal properties of liquids and gases. The lbm is the amount of mass which weighs 1 lbf under standard conditions (at a latitude of 45° and at sea level). In order to avoid the confusion which would be caused by the use of two units for mass (slug and lbm), in this textbook we use almost exclusively the unit slug for mass. This practice makes dynamics much simpler than if the lbm were used. In addition, this approach allows us to use the symbol lb to always mean pound force.

Additional quantities used in mechanics and their equivalent base units will be defined as they are introduced in the chapters which follow. However, for convenient reference these quantities are listed in one place in Table D/5 of Appendix D.

Professional organizations have established detailed guidelines for the consistent use of SI units, and these guidelines have been followed throughout this book. The most essential ones are summarized in Table D/5 of Appendix D, and you should observe these rules carefully.

1/5 Gravitation

Newton's law of gravitation, which governs the mutual attraction between bodies, is

$$\boxed{F = G\frac{m_1 m_2}{r^2}} \qquad (1/2)$$

where $F =$ the mutual force of attraction between two particles

$G =$ a universal constant called the *constant of gravitation*

$m_1, m_2 =$ the masses of the two particles

$r =$ the distance between the centers of the particles

The value of the gravitational constant obtained from experimental data is $G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$. Except for some spacecraft applications, the only gravitational force of appreciable magnitude in engineering is the force due to the attraction of the earth. It was shown in *Vol. 1 Statics*, for example, that each of two iron spheres 100 mm in diameter is attracted to the earth with a gravitational force of 37.1 N, which is called its *weight*, but the force of mutual attraction between them if they are just touching is only 0.000 000 095 1 N.

Because the gravitational attraction or weight of a body is a force, it should always be expressed in force units, newtons (N) in SI units and pounds force (lb) in U.S. customary units. To avoid confusion, the word “weight” in this book will be restricted to mean the force of gravitational attraction.

Effect of Altitude

The force of gravitational attraction of the earth on a body depends on the position of the body relative to the earth. If the earth were a perfect homogeneous sphere, a body with a mass of exactly 1 kg would be attracted to the earth by a force of 9.825 N on the surface of the earth, 9.822 N at an altitude of 1 km, 9.523 N at an altitude of 100 km, 7.340 N at an altitude of 1000 km, and 2.456 N at an altitude equal to the mean radius of the earth, 6371 km. Thus the variation in gravitational attraction of high-altitude rockets and spacecraft becomes a major consideration.

Every object which falls in a vacuum at a given height near the surface of the earth will have the same acceleration g, regardless of its mass. This result can be obtained by combining Eqs. 1/1 and 1/2 and canceling the term representing the mass of the falling object. This combination gives

$$g = \frac{Gm_e}{R^2}$$

where m_e is the mass of the earth and R is the radius of the earth.* The mass m_e and the mean radius R of the earth have been found through experimental measurements to be $5.976(10^{24})$ kg and $6.371(10^6)$ m, respectively. These values, together with the value of G already cited, when substituted into the expression for g, give a mean value of $g = 9.825\ \text{m/s}^2$.

The variation of g with altitude is easily determined from the gravitational law. If g_0 represents the absolute acceleration due to gravity at sea level, the absolute value at an altitude h is

$$g = g_0 \frac{R^2}{(R + h)^2}$$

where R is the radius of the earth.

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*It can be proved that the earth, when taken as a sphere with a symmetrical distribution of mass about its center, may be considered a particle with its entire mass concentrated at its center.

Effect of a Rotating Earth

The acceleration due to gravity as determined from the gravitational law is the acceleration which would be measured from a set of axes whose origin is at the center of the earth but which does not rotate with the earth. With respect to these "fixed" axes, then, this value may be termed the *absolute* value of g. Because the earth rotates, the acceleration of a freely falling body as measured from a position attached to the surface of the earth is slightly less than the absolute value.

Accurate values of the gravitational acceleration as measured relative to the surface of the earth account for the fact that the earth is a rotating oblate spheroid with flattening at the poles. These values may be calculated to a high degree of accuracy from the 1980 International Gravity Formula, which is

$$g = 9.780\,327(1 + 0.005\,279 \sin^2 \gamma + 0.000\,023 \sin^4 \gamma + \cdots)$$

where γ is the latitude and g is expressed in meters per second squared. The formula is based on an ellipsoidal model of the earth and also accounts for the effect of the rotation of the earth.

The absolute acceleration due to gravity as determined for a nonrotating earth may be computed from the relative values to a close approximation by adding $3.382(10^{-2}) \cos^2 \gamma$ m/s^2, which removes the effect of the rotation of the earth. The variation of both the absolute and the relative values of g with latitude is shown in **Fig. 1/1** for sea-level conditions.*

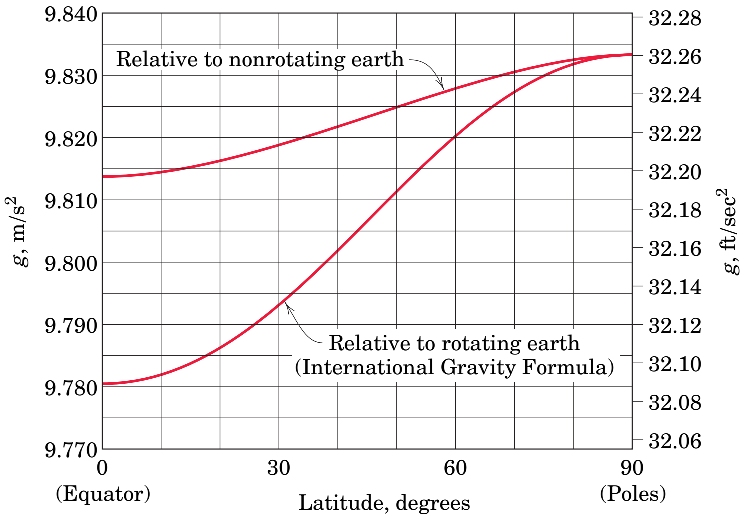

FIGURE 1/1

Standard Value of *g*

The standard value which has been adopted internationally for the gravitational acceleration relative to the rotating earth at sea level and at a latitude of 45° is 9.806 65 m/s^2 or 32.1740 ft/sec^2. This value differs very slightly from that obtained by evaluating the International Gravity Formula for $\gamma = 45°$. The reason for the

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*You will be able to derive these relations for a spherical earth after studying relative motion in Chapter 3.